FLIGHT DYNAMICS OF SAILING FOILERS

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Abstract. This paper explores the application of the classic methods of flight dynamics to the passive stability of a sailing foiler. The equations of motion are developed. Their eigenvectors give the natural modes which, even if not exact, provide a better basis for dynamical studies than the Cartesian coordinates.
Parametric studies show the variation of the natural modes with some design and trim parameters. Unlike aircraft, whose set of natural modes is usually consistent over the design space, the sailing foiler shows a continuous variation of mode characteristics.
The goal of foil design is to maximize performance with acceptable handling. The paper discusses the effects of frequency, damping and mode shape on seakeeping and handling, and so indicates the effect of some design and trim parameters on controllability. Some trends emerge which agree with experience.

1. NOMENCLATURE

DOF degree of freedom
CG Centre of Gravity
X,Y,Z force components on body axes
L,M,N moment components on body axes

{u,v,w} velocities in body axes
{p,q,r} angular velocities about body axes

$\boldsymbol{x}$ vector of displacements
$\boldsymbol{v}$ vector of velocities
x displacement on X axis
y displacement on Y axis
z displacement on Z axis

α incidence in XZ plane
β incidence in XY plane
γ incidence in YZ plane
ζ Damping ratio: damping expressed as a proportion of the critical damping.

Cl lift coefficient
Cd drag coefficient
Cly sideforce coefficient
Clz vertical lift coefficient
$K_{i,j}$ Lift-incidence slopes
$Cd0$ drag coefficient at zero incidence
$d12$ drag coefficient factor for α
$d13$ drag coefficient factor for β
S a reference area
q dynamic pressure
C a control vector

2. INTRODUCTION

2.1 Background and history

The discipline of flight dynamics grew with the aircraft industry. It was pioneered in the early 19th century notably by the Wright brothers, Lanchester, and Glauert. By the 1940s the theory and methods were well established and a large body of experimental results enabled the design of aircraft with known performance, stability and handling qualities. The discipline is well documented. For instance [1] provides a summary of the state of the art at the time. Etkin provides clear portrayals in his successive publications [2] Data for practical design may be obtained from [3] or more recently from CFD. Applications such as AVL [4] permit the rapid investigation of the performance and stability of different aircraft configurations.

2.2 Foilers

Several prototype foil-born sailing craft [5] were built between the 1930s and the 1980s, but it was in the 2000s that foilers gained prominence with the International Moth and then the AC72. There are now several sailing foilers in production.
Published work e.g [6] [7] on sailing foilers tends to concentrate on performance, with excursions into single-DOF static stability. Furrer [8] developed a 6-DOF model for performance estimates which could be extended to cover dynamic stability.

Typically static stability estimates are limited to pitch and roll stability. Pitch stability is estimated by consideration of the lift-curve slopes of the foils. Roll stability is estimated using geometric considerations. These methods are at best approximate because they assume a single-DOF natural mode. Furthermore they make no estimates of natural frequency or damping rate.

In the present publication we develop the equations of motion of a sailing foiler in the same way as for an aircraft, including mass, damping and stiffness terms. The resulting equations are first solved for equilibrium to yield the

1 Independent Engineer

attitude and control settings for steady flight. This step is equivalent to a 6-degree-of-freedom VPP.
Subsequently, the motion following a perturbation from the equilibrium state may be traced numerically, but more insight is gained by extracting the natural modes and frequencies from the linearized equations. The effects of a perturbation can then be evaluated by combination of the natural modes. This is preferable to working with the Cartesian coordinates because the natural modes are orthogonal.

The advantage of the former approach is that it captures the nonlinearities inherent in foil behaviour. The advantage of the latter is the insight it gives into the motion

3. THEORY

3.1 Stability

In the theory of flight, as with any dynamical system, it is usual to differentiate between static and dynamic stability.
A mode is said to be statically stable if following a perturbation it tends to return to its original state. A mode is said to be dynamically stable if the oscillations following a perturbation tend to decay rather than grow. (Figure 2)
Of course different natural modes of a system can have different stability characteristics.

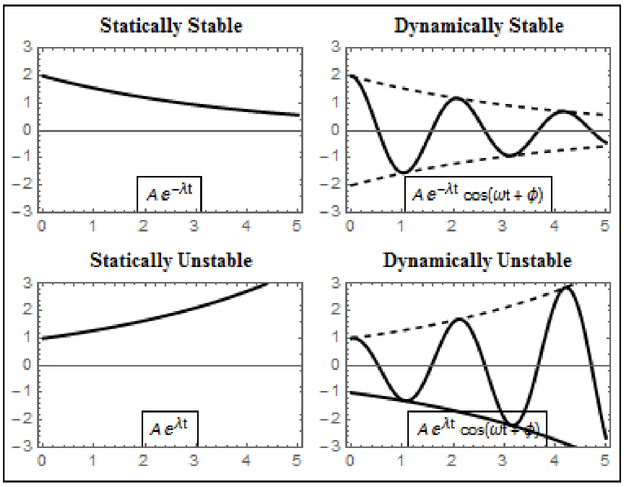
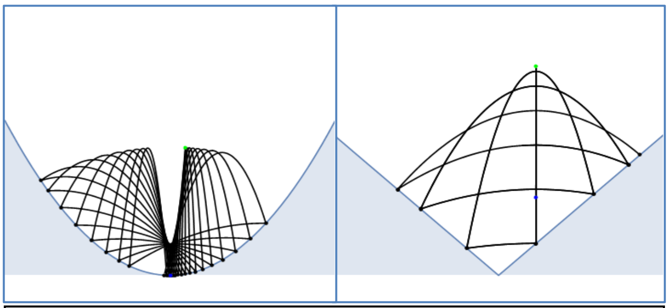


Figure 2 Linear stability

A system may be active or passive. An active system is one in which a control system provides guidance in response to measurements of the state. In a passive system there is no such control.
A system may be linear or non-linear. In a linear system, coefficients such as mass, stiffness and damping – and the properties of any control system – are constant. Linear systems have closed solutions and relatively simple behaviour.. The motion is either a cycle or an exponential, or both.
Even simple nonlinear systems can have complex behaviour. In general (Figure 1) they never settle to a constant pattern.

Aircraft are usually designed to have linear aerodynamic



A ball bouncing in a trough is a simple non-linear system which in general (left) shows complex behavior. In special conditions (right) it can settle to a repeating pattern. This is termed a *limit state cycle*.
(Modified from [14])

Figure 1 A nonlinear system

characteristics over most of their operating envelope. This makes the flight characteristics deterministic. Even if the aerodynamic behaviour is not linear it is usually smooth enough to make linearization valid for small excursions. This allows linear methods to be used to predict stability and response to typical perturbations.

Sailing foilers are rarely linear. In surface-piercing configurations (Figure 3) the immersed area varies non-linearly with height. Waves typically have amplitude greater than the foil dimensions.
However, the linear approach exposes behaviour such as resonance. Knowledge of the systems natural frequencies is important to avoid the wave-approach frequency as well as the frequencies of structural vibration. Knowledge of the natural modes and their damping factors helps the development of technique and of control systems. It also helps with analysis of logged data. It is easier to interpret in eigenvector space.

If the vehicle is marginally stable for small perturbations it is unlikely to be better for large ones. The linear study can give good indications on design trends, and is a base for further studies such as seakeeping.

3.2 Typical aircraft characteristics

The equations of motion have 12 roots but this does not mean 12 independent modes. A cyclic mode has a conjugate pair of roots. There are also at least 3 ‘zero’ roots. Most modern aircraft have 3 cyclic modes and one divergent mode, with the remainder being stable and convergent.

The symmetrical cyclic modes are known as short-period pitch and phugoid.
Short-period pitch is a pitch oscillation about the centre of gravity with the other ordinates changing little.

Phugoid is a cyclic interchange between speed and height resulting in an undulating flight-path at almost constant incidence.
The antisymmetric modes are known as spiral divergence, and Dutch roll.
Dutch roll is an precession with fairly equal components of pitch, roll and yaw out of phase. The phase difference is about $\pi/2$. Seen from the cockpit, the wingtip appears to circle. This mode usually has positive damping.
The spiral mode is a divergence combining roll, sideslip, and increasing airspeed. Most aircraft will spiral if not controlled.

3.3 Dynamical systems

The flight dynamical system typically contains mass, damping and stiffness effects. The force balance may be written as:

$$\boldsymbol{F}(\boldsymbol{x}, \dot{\boldsymbol{x}}, \dot{\boldsymbol{x}}, \ldots, \boldsymbol{C}) = \boldsymbol{Fe}(\boldsymbol{t}) \quad \textbf{(1)}$$

Where $\boldsymbol{x}$ is a vector of parameters, C is a control vector, Fe(t) is an external force and F contains the transfer functions of force vectors due to the parameters and their successive time derivatives.
For stability analysis the equations (1) are expanded as truncated Taylor series taking Fe(t) = 0, yielding the linear form

$$\boldsymbol{M}.\dot{\boldsymbol{x}} + \boldsymbol{B}.\dot{\boldsymbol{x}} + \boldsymbol{K}.\boldsymbol{x} + \boldsymbol{Fe}(\boldsymbol{t}) = \boldsymbol{0} \quad \textbf{(2)},$$

where M, B and K are square matrices respectively of Mass, Damping and Stiffness.

3.4 The Classical Aircraft Method

In aircraft dynamics, the parameter vector *x* contains the 3 rigid-body translations and 3 rigid-body rotations, followed as needed by terms for structural deflections, controls, engine thrust, etc. The equations are usually developed in an axis set tied to the mean flight-path (the stability axes frame) because that simplifies the equations. We are initially only concerned with the rigid-body ordinates.
The linear equations of motion are built up using the set of *aerodynamic derivatives*.

Stiffness terms (collected into matrix K)

$$[X|Y|Z|L|M|N]_{[u|v|w|p|q|r]}$$

Damping terms (collected into matrix B)

$$[X|Y|Z|L|M|N]_{[\dot{u}|\dot{v}|\dot{w}|\dot{p}\ |\dot{q}|\dot{r}]}$$

Mass terms (collected into matrix M)

$$[X|Y|Z|L|M|N]_{[\ddot{u}|\ddot{v}|\ddot{w}|\ddot{p}|\ddot{q}|\ddot{r}]}$$

where, for example $N_{\dot{p}}$is the derivative of yawing moment with respect to roll rate.

Much experimental work [3] has gone into tabulating these derivatives, although now they are often calculated using panel methods.

The equations (2) have a solution of form

$$\boldsymbol{X} = \sum\left(\boldsymbol{X0}_i e^{\lambda_i t}\right) \quad \textbf{(3)},$$

where the $\mathbf{X0}$ and $\boldsymbol{\lambda}$ are the 12 complex eigenvectors and 12 complex eigenvalues.

Most aircraft are symmetrical. Thus the symmetric and anti-symmetric motions are uncoupled and can be considered independently as 2nd-order, 3 DOF systems. Furthermore, some terms disappear because of the choice of stability axes and assumptions such as buoyancy and gravity being independent of height, conveniently yielding two independent 2nd order systems in 2 degrees of freedom each. These systems yield 4th-order characteristic polynomials and so can be solved directly. This fact no doubt influenced the standard choice of assumptions.

3.5 Differences with foiler methods

To model the dynamics of a sailing foiler we also build up and solve equations (1) and (2). The method is different in detail because the foiler has several characteristics which distinguish it from an aircraft;

- It operates at the water/air interface. Some components are in air and some in water, with different velocities.
- There is a requirement to maintain a given ride height, so foil systems are designed to provide stiffness in the vertical direction, either by their geometry or by a control system. There are only two zero roots, one less than for an aircraft. The linear system has 10 roots.
- The system is not symmetric.
- The steady flight condition is more complex to identify because it involves finding equilibrium on all 6 axes.
- There is no published body of experimental data which can be used to derive the aero -(and hydro)-derivatives.

3.6 The Foiler Model

We consider each foil to act at a point, generating forces and moments which are functions of its attitude to the local flow and of its immersion $\boldsymbol{z}$. The foils are fixed in the body frame. Their orientations in the body frame may be used as control variables. The sail-plan in the present study is rigid and is modelled using strip theory without wind gradient.

Two foil representations were used. For design-space explorations the foils may be modelled minimally as:

$$\begin{pmatrix} Cly \\ Clz \end{pmatrix} = \begin{pmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \end{pmatrix} \cdot \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \quad (4)$$

$$Cd = Cd0 + d12 \cdot Cdy^2 + d13 \cdot Cdz^2 \quad (5)$$

with
$S(z) = S0 + k(z - z0)$(k is a heuristic function)
and

$$\begin{pmatrix} Fx \\ Fy \\ Fz \end{pmatrix} = q \cdot S(z) \cdot \begin{pmatrix} Cd \\ Cly \\ Clz \end{pmatrix} \quad (6)$$

This model is useful for gaining insight, and is adequate for a submerged T-foil and for the sail plan. It is too simple to represent the nonlinear behaviour of a surface-piercing foil. In the present study these foils are represented by interpolation from a lookup table.

3.7 Numerical Procedures

The model is implemented in *Mathematica*. [9] Body coordinates are used. The position and attitude of each foil is defined by its body-frame ordinates. Its world-frame position and attitude are derived by applying the geometric transformation due to the position and attitude of the platform. The forces on the foil are then calculated from its height and attitude, transformed back to the body frame and summed into the overall force balance of the platform.

The first step is to find the equilibrium condition. We solve numerically for a control vector holding the attitude constant

$$\begin{pmatrix} X \\ Y \\ Z \\ L \\ M \\ N \end{pmatrix} = F1 \begin{pmatrix} V0 \\ leeway \\ z = ht \\ elevator \\ rudder \\ rigangle \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{st} \begin{pmatrix} x \\ y \\ \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (7)$$

The secant method of solution is used. It is often necessary to provide the starting point by hand.
The resulting control vector for equilibrium is then inserted into equations (1) to give the platform force balance in the form:

$$F(x, \dot{x}, \dot{x}, C) = 0 \quad (8)$$

Three methods of solution are then available:
Firstly, direct numerical integration from an initial perturbation using the Mathematica function **NDSolve**. This function can equally be used to trace the response of the nonlinear equations.
Secondly, differentiation of **F** wrt **X** and its first and second derivatives yields the matrices M, B and K to substitute into equations (2). This 6-DOF, 2nd order system is converted to a 12-DOF, 1st order system [10], [11] permitting the Eigen-solution to be found.
Thirdly, as a check the 6-DOF, 2nd order linear system may be solved directly using the Mathematica function **DSolve**.

The model was checked using estimated data for a Supermarine Spitfire Mk XIV. The geometry and the flight characteristics are available (although the load condition is not clearly stated), but the mass distribution was estimated on the basis of the few data available. Nevertheless the natural modes were correctly identified and the phugoid and Dutch roll frequencies were within 33% of measured data. This is considered acceptable given the quality of the data and the simplicity of the strip-theory wing model.. The short-period pitch and spiral divergence rates are very sensitive to the weight distribution and so no conclusions can be drawn.

4. NUMERICAL EXPERIMENT

4.1 The Model

The numerical experiments were performed using data representing Groupama C, the current (2013) world champion C-Class catamaran.

Figure 3- Groupama C

The configuration consists of a main 'J' foil amidships to leeward with inverted 'T' foils near the rear of each hull. The main foil to windward is always raised.
The panel code AVL [4] was used for the main foil to provide tabular data for the hydrodynamic forces against immersion depth and attitude. In the model these data were interpolated with piecewise cubic splines.
AVL cannot model the free surface directly. Two models were created, one modelling the water surface as a rigid wall Figure 4 , and one modelling it as an anti-reflection Figure 5. Comparison with RANS has apparently shown that, the solid-surface model is over-stiff and the anti-reflection model is more accurate

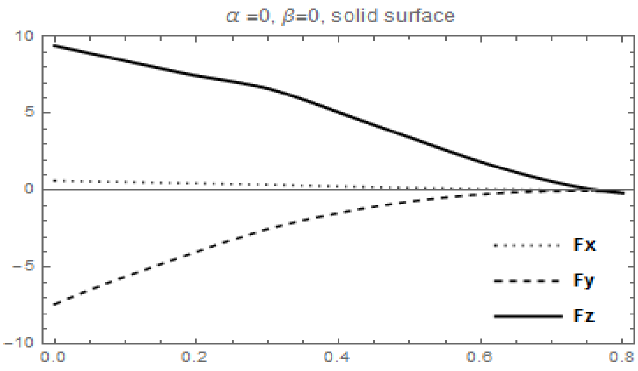


Figure 4 Foil Model – solid surface

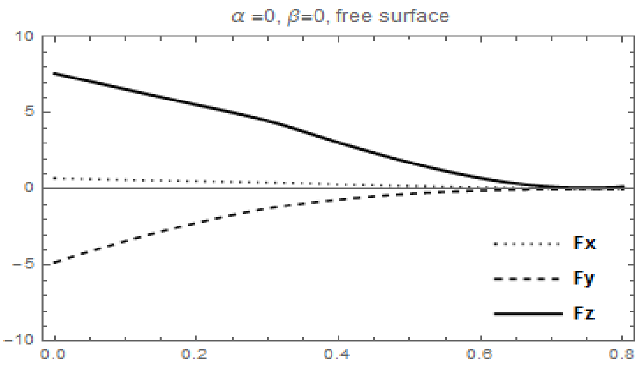


Figure 5 Foil Model –anti-reflecting surface

The rig was modelled using strip theory. This is considered acceptable for a first instance because the foils contribute a large proportion of the stiffness and damping

4.2 Procedure

The procedure described in [3.7] was run about 250 times with different model parameters. These were:

- Centre of gravity position fore/aft;
- Horizontal tail-foil area
- Incidence of main foil
- Lift-incidence slope of the rig.

The true wind speed and direction were specified to represent a downwind leg with 30-30 knots boat speed. For each configuration the (complex) eigenvectors and eigenvalues were recorded along with the model description

5. RESULTS

5.1 General characteristics

This model usually shows three cyclic modes and four convergent modes. In some configurations (see *Rig lift-incidence slope*) two of the cyclic modes merge into a pair of non-cyclic modes.

Usually, all but one of the non-cyclic modes is strongly damped. The last is usually either weakly damped or divergent.

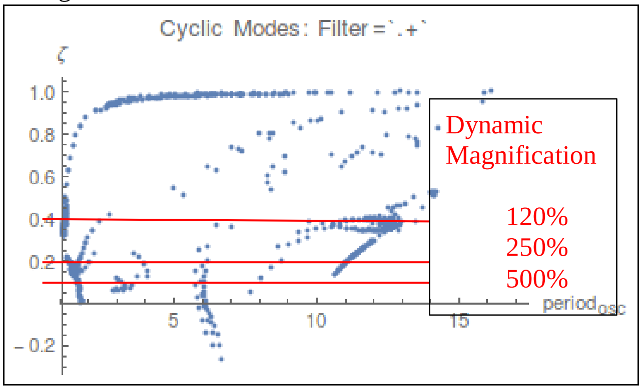


Figure 6 Damping ratio vs Frequency

[Figure 6] is a summary of all the cyclic modes. It is seen that about half of the configurations show strong (> 30% critical) damping. There are many points with low enough damping ratio to amplify perturbations by several 100%. There are some configurations which show negative damping. These were some fairly symmetrical modes associated with small tail area and aft CG position. Figure 10 to Figure 14 (Video clips posted at http://www.openfsi.org/doku.php?id=data:index show the motion more clearly.) (In the figures, the arrows show velocity) The modes do not fall into categories as cleanly as those of an aircraft. This is due to asymmetry. The mode shapes show a fairly continuous progression from nearly-symmetric to nearly anti-symmetric.

<u>pitch and heave</u>

No equivalent of the short-period pitch was observed. In its place was observed a mode with mainly pitch and heave.

<u>near-phugoid</u>

In the near-symmetrical modes there can be a near-phugoid mode, although more pitch amplitude is observed than for an aircraft. There is always some anti-symmetric component

<u>Diagonal pitch</u>

This mode consists mainly of pitch and yaw in phase, giving the impression of a cyclic rotation in a plane at 45^o to the water surface.

<u>Dutch-roll</u>

The classic precession was observed.

<u>skate</u>

A mode dominated by sideslip with some yaw.

<u>Fast yaw</u>

A mode dominated by yaw.

There is no clear ranking of the modes by frequency.

Where a non-cyclic mode was divergent, the mode shape corresponded to the classical sailboat broach, combining yaw to windward, acceleration and pitch-up. Fortunately it is easily corrected by the rudder.

5.2 Parametric studies

5.2.a Rig lift-incidence slope

The lift-incidence slope of the rig depends on its stiffness and so is easily modified. This high-aspect-ratio rig would have dCl/dα about 90% 2π if rigid. Figure 7 and Figure 8

show a sweep made increasing the rig dCl/dα from 2.5 (corresponding to elastic twist of about 10^{o}) to 6.25.
This series shows a marked change in the values of the roots which correspond to the precession-type mode.

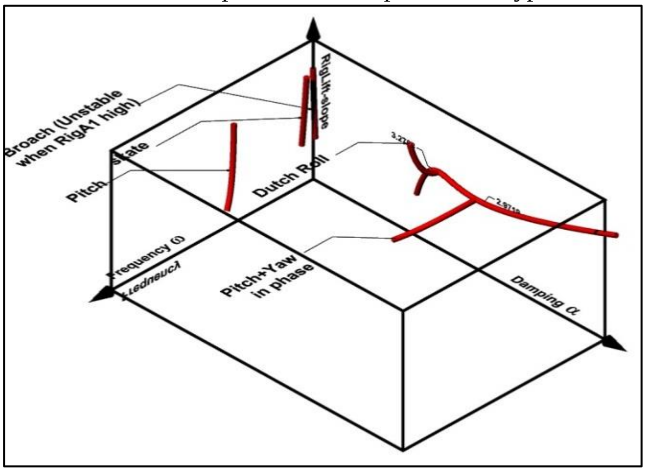


Figure 7 Effect of dCl/dα

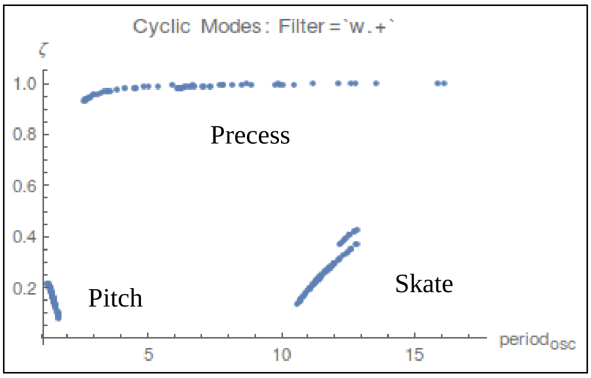


Figure 8 Effect of dCl/dα

For low dCl/dα, the mode is a diagonal pitch with high damping.. As dCl/dα increases, the damping increases a little and the frequency reduces to zero. For a small range of dCl/dα (3 to 3.25) the oscillation disappears. With further increase of dCl/dα a precessive mode emerges. As dCl/dα increases further, the damping increases, as does the phase difference between pitch and roll;
The coupling from the rig between symmetric and anti-symmetric motion is approximately orthogonal to that from the foils. Thus the principle axes of the damping matrix are sensitive to the amount of damping from the rig, which is proportional to its dCl/dα.
When its dCl/dα is low the foils dominate mode shape. When it is high the rig dominates.

5.2.b Horizontal Stabilizer area and CG position

A two-dimensional sweep Figure 9] was made varying the area of the horizontal stabilizer, and the fore/aft position of the center of gravity.

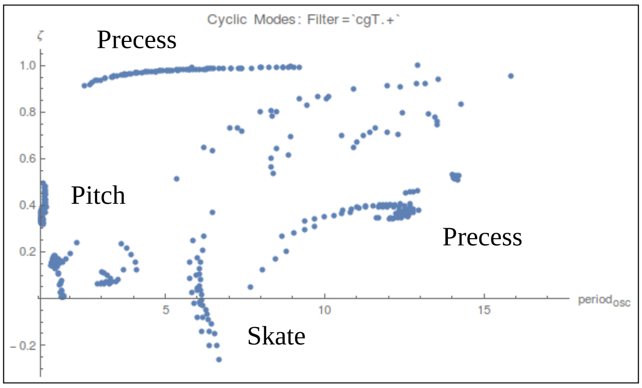


Figure 9 Tail area and CG position

With the CG well aft the Skate mode becomes dynamically unstable. This is not surprising because the transfer of load to the tail foils reduces the area of the main foil
Reducing the tail foil area to about 20% of design size generally reduces the damping, especially in the pitch mode.

The frequencies of the near-phugoid and pitch/heave modes rise initially with decreasing tail area and then drop fast. Their damping decreases with decreasing tail area. For further reduction in area, it might be expected that the mode's damping becomes negative. But instead, we observe that the cyclic mode disappears, to be replaced by a convergent, monotonic mode.
Skate Mode is a little affected by tail area because it is very anti-symmetric.

5.3 Discussion

[Figure 6] shows that no configurations were observed in which all the damping ratios were above about 30%. That corresponds to about 200% of dynamic amplification. There are usually some modes which give dynamic amplification of several 100's of a %.
Thus, within the parameter range investigated there is always a perturbation which can give rise to large amplitude motion if excited near one of the natural frequencies. We have seen that the cycle periods range from 20 seconds to less than 1. The higher frequencies approach the natural frequencies of the structure, leading to resonance. The middle frequencies may coincide with the surface wave frequency, also leading to resonance.

Whilst there is some design freedom to choose which of the modes will be livelier, it would be difficult to ensure that all modes are manageable. This has immediate consequences for oceanic foiling.
The fastest control is the steering which is purely anti-symmetric. It is effective in controlling a yawing motion like the Skate mode even if resonant or unstable. But steering can not directly affect symmetrical motion. (For that, some means of DLC (direct lift control) would be required.) In addition, a sailing foiler must maintain the ride height within close limits.

The rig provides a large damping component which provides symmetric/anti-symmetric coupling. The rig dCl/dα sweep showed that the damping from the foils has rather different characteristics. By considering the two together it may be possible to increase symmetrical damping.

Awareness of the sensitivity of the boat in terms of its natural modes and frequencies will enhance safety: If the crew is familiar with the resonant frequencies, it becomes possible to choose a course to avoid them. For aircraft pilots, understanding of the natural modes is essential for good technique. Foiler pilots would benefit similarly, because this knowledge makes it easier to understand what is happening and to make the right judgments.

6. CONCLUSIONS

We have demonstrated that the classical techniques of flight dynamics can be applied to sailing foilers to obtain the natural modes.

The popular 'J' foil configuration often has three lightly-damped cyclic modes: one similar to phugoid, one similar to Dutch roll, and one combining side-slip and yaw that we name 'skate'. It also exhibits the well-known sailboat broaching behaviour However unlike an aircraft whose mode shapes are distinct, the mode shapes are sensitive to the configuration. We believe that the variation of mode shapes is due to the high (and variable) coupling between symmetric and antisymmetric motion. Some configurations were identified as dangerous. As for an aircraft, we find that moving weight aft is de-stabilizing.

In this example, the frequencies of the first two modes are often fairly close, to the extent that some parameter changes cause them to merge. Some parameter sweeps cause them to cross over.
The models of the individual foils were relatively simple, yet the trends in the calculated modes seem reasonable. Clearly, the same procedure could be used with improved foil and rig modelling.
No dynamic instabilities were detected in the baseline model, which reflects the performance of Groupama C.

Knowledge of the natural modes and their trends is useful for development both of design and of technique. Multi-dimensional parameter sweeps provide additional insight once the software ergonomics have been mastered.
Working (and thinking) in eigenvector space is a powerful aid to understanding because it is orthogonal. This simplifies the design of control strategies.
This paper presents neither an exhaustive design study nor even a fully-operational method. The modelling of individual foil and rig response is oversimplified. It is a base on which to build.

Acknowledgements

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8. FIGURES

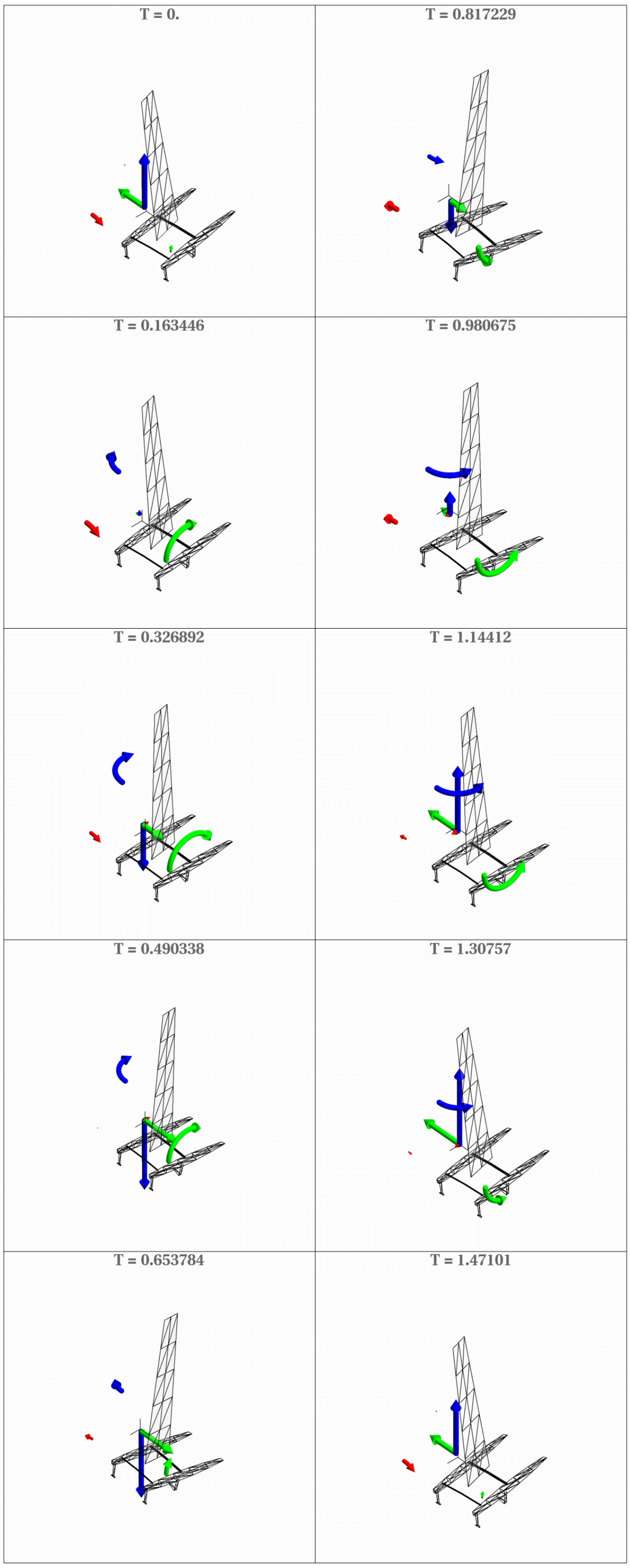

Figure 10: Near-phugoid mode. In the near-symmetrical modes there can be a near-phugoid mode, although more pitch amplitude is observed than for an aircraft. There is always some anti-symmetric component

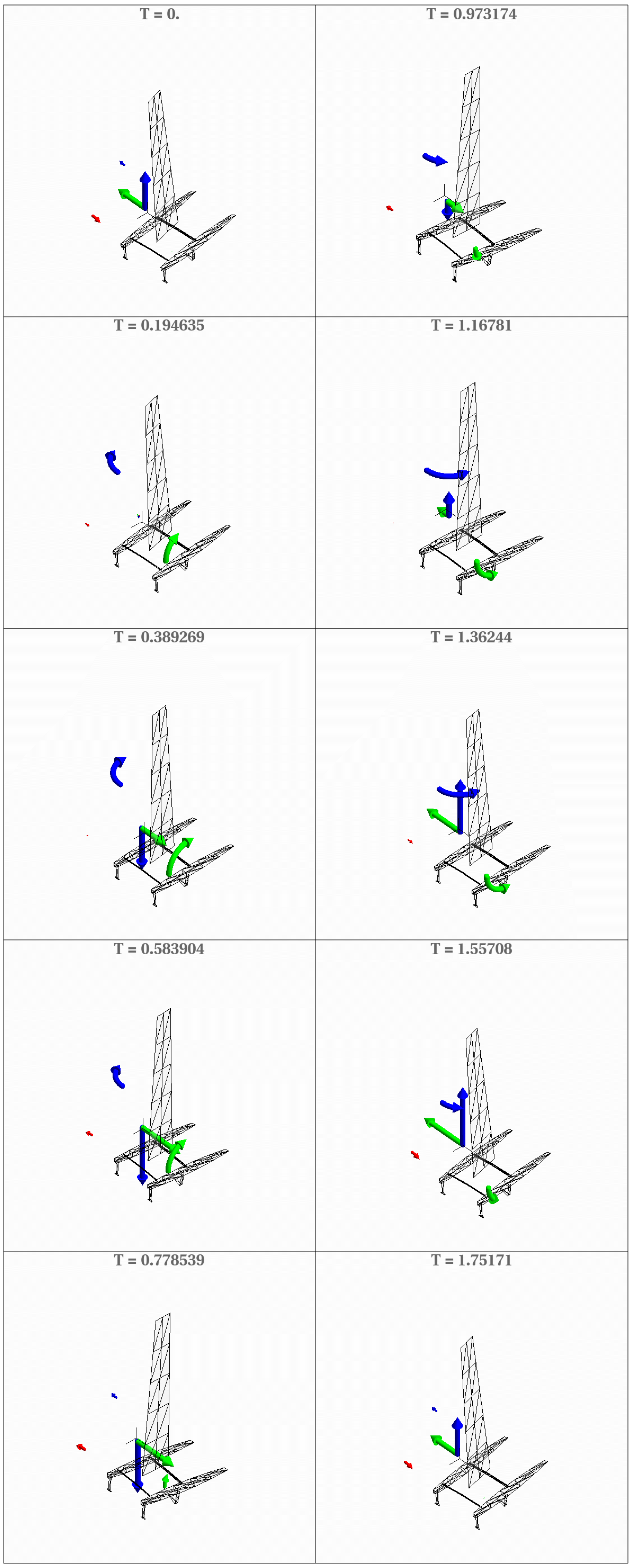

Figure 11: Diagonal Pitch mode. This mode consists mainly of pitch and yaw in phase, giving the impression of a cyclic rotation in a plane at 45o to the water surface.

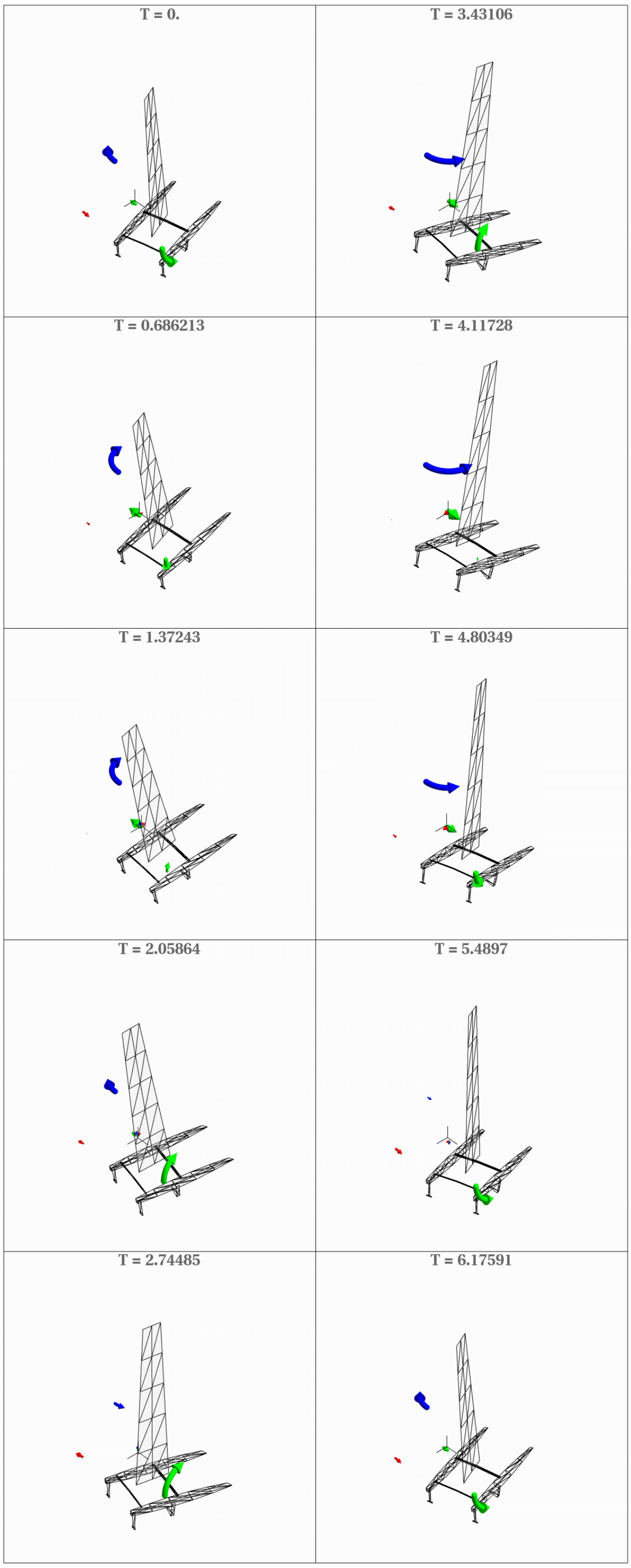

Figure 12 Dutch Roll mode. The classic precession

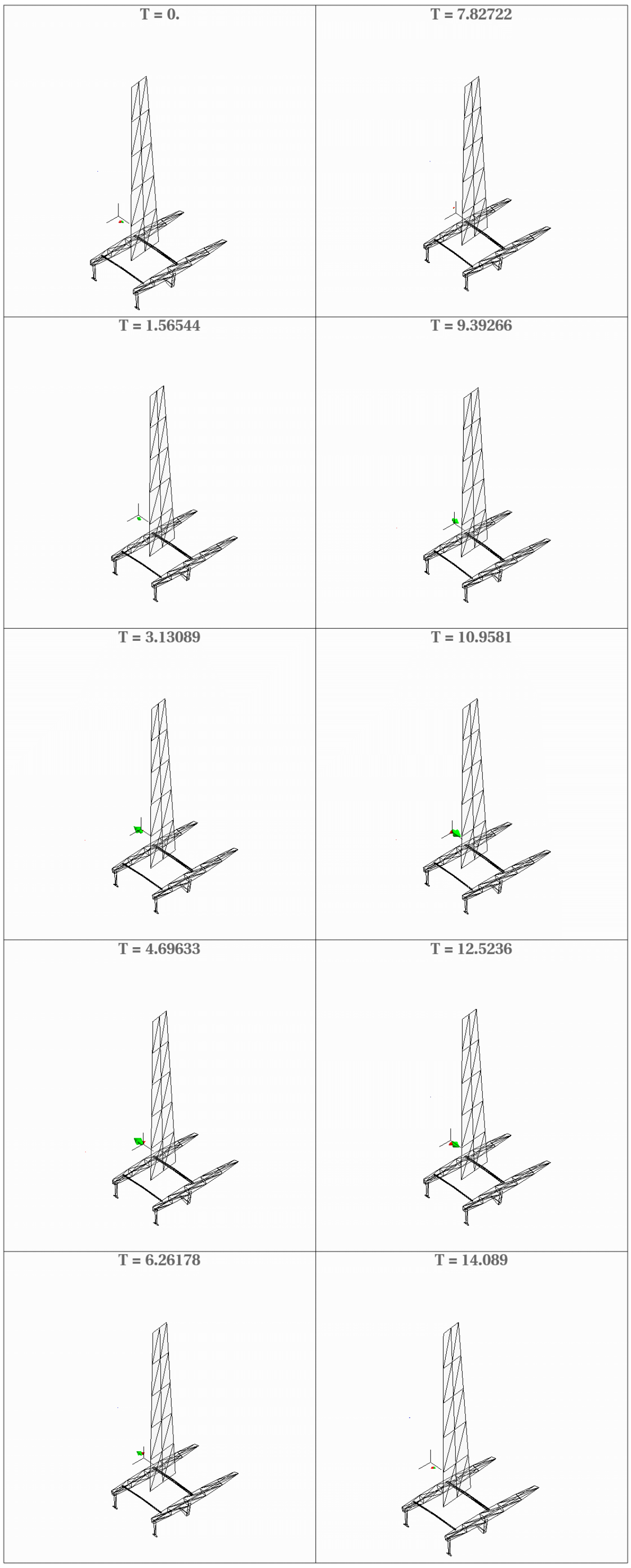

Figure 13 Skate mode: A mode dominated by sideslip with some yaw.

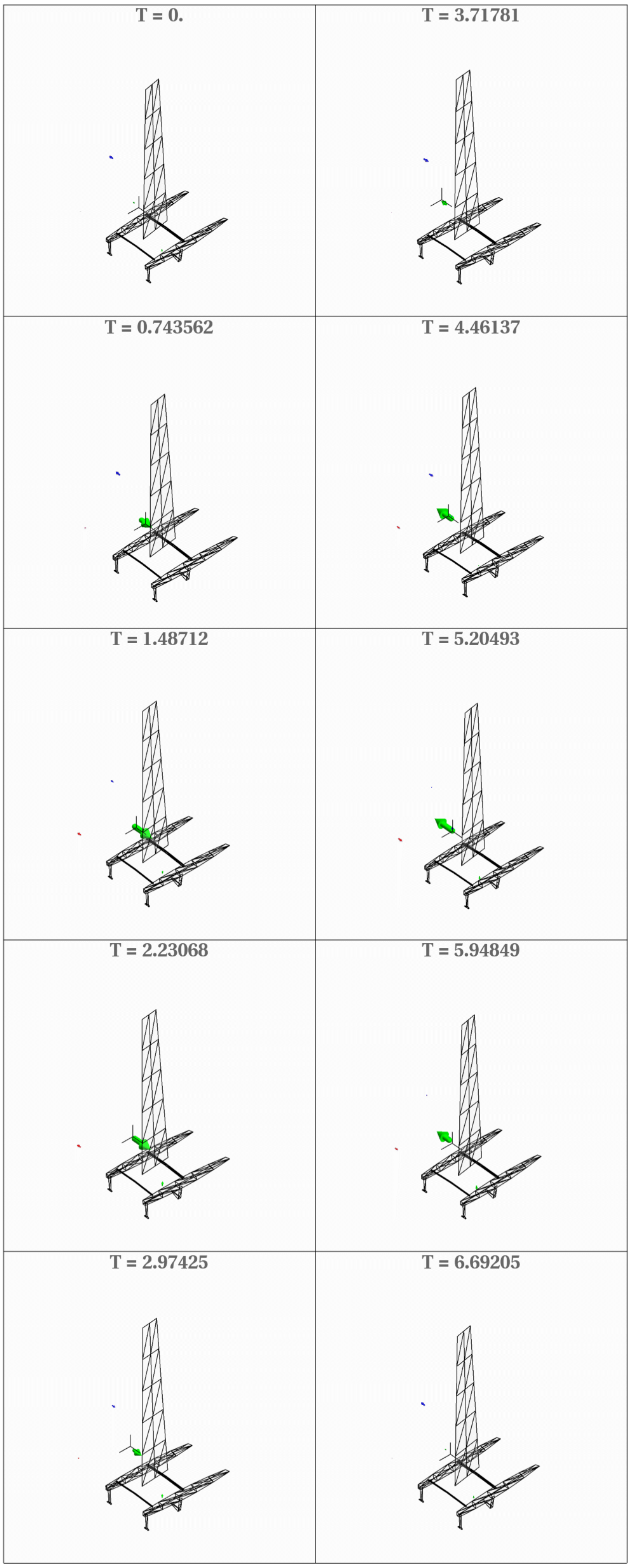

Figure 14.: Fast Yaw mode. This was the only mode observed with negative damping. It occurs when the centre of gravity is aft and the tail foils are small.