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RESEARCH ARTICLE

Determination of latitude by two fixed-altitude sightings

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Abstract

The use of multiple observations near noon with a traditional sextant to determine a fix is common among celestial navigators. A recent invention is the fixed-angle 'Bris sextant' that comes with advantages, but imposes constraints due to its invariant nature. We propose a method by which both longitude and latitude can be fixed using only two sightings with such a device, each equidistant from the meridian. By modelling the solution space for the method, we explore some of the potential utility across geography and seasonal variation. Although this method was developed for use with a Bris fixed-angle sextant, it can also be conveniently used with a more traditional marine or level-bubble sextant. Because this method is computationally cumbersome, it is most convenient when used in a computer or tablet application, or with tables.

1. Introduction

The use of multiple sightings of a single celestial object close to meridian passage for determining latitude is a long-known technique (e.g. latitude by two altitudes or two ex-meridians – Lax, 1799; Bowditch, 1802; Moore, 1807; Mackay, 1810; Raper, 1833; Norie, 1835; Taylor, 1837; Johnson, 1895; Thoms, 1902; Cugle, 1922; Weems, 1940; Silverberg, 2012; and see Cotter, 1964 for a review). This has found particular utility in daytime sightings of the Sun when sightings are taken just prior, during and after local noon (Galton, 1878; Muir, 1911). The general utility of using the Sun for latitude means that it continues to be used by open-water sailors, who will often take the noon sights with a sextant to confirm position even in the age of GPS technology.

2. The Bris sextant

The modern celestial sextant is a highly refined instrument and has the theoretical potential to locate a navigator within hundreds of yards. However, it is an expensive and delicate tool that must be used and stored with care. Further, it requires considerable training and practice to use well. An alternative has been created by Swedish boatbuilder and sailor Sven Yrvind (Yrvind, 2008). This is known as the 'Bris sextant' and is a simple optical device with no moving parts, being composed of a few (two or three typically) small pieces of glass epoxied together at slight angles (Figure 1). It produces a series of reflections of the celestial body (typically the Sun) that are at fixed angles below the Sun. When one of the reflections is superimposed over the horizon, the Sun is at that pre-determined angle above the horizon (Figure 2). Although the device is simple, it possesses a few advantages over traditional sextants. The first is cost – good nautical sextants can run well over US$1500 (as of 2022), but a Bris sextant can be made for a few dollars (they are typically built by the user and inexpensive medical/biological slides are often used). The second is durability – if small and well-epoxied, they are very durable. However,

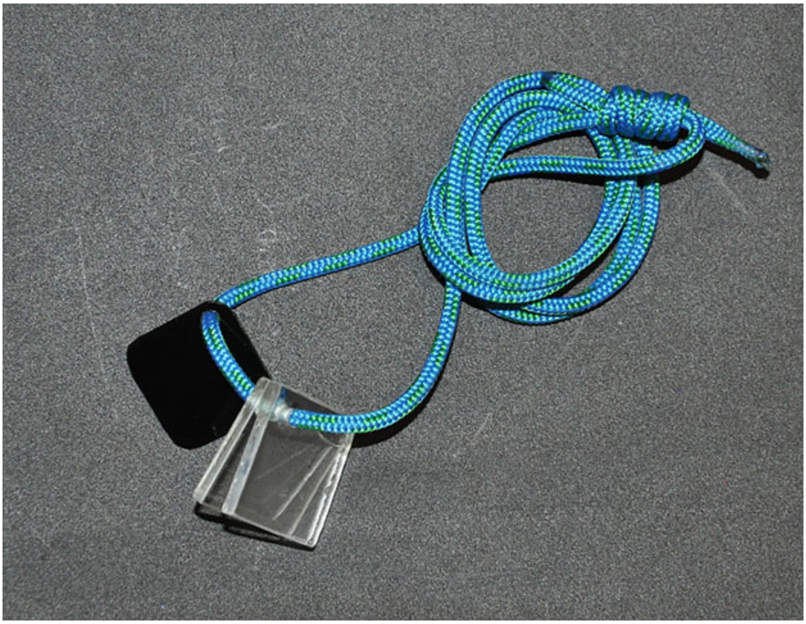

Figure 1. *A Bris sextant. This is a simple model with only two panes of glass, drilled to be carried on a lanyard around the neck (for quickly checking the Sun). In this case, greater refraction is made possible with the use of teleprompter-type glass which makes more of the light refract rather than pass through. Note that this model also has a shade-5 welding filter on the lanyard to prevent the brighter images from damaging the viewer's eyes. Higher images, which are only refracted a few times, can be quite bright, whereas lower images are dimmer.*

perhaps the most valuable feature is that, even as the Bris sextant moves in front of the eye, the image stays fixed as the sum of the angles is maintained and the net refraction does not change. In this, it is similar to a 'reflex-style' gunsight in which the target can move with the sight and slight shifts do not affect the relationship of the optical targeting reticle to the target itself. In a moving ship or boat, this is of considerable value. Here we propose a new method of celestial fix using the Bris sextant to determine latitude and longitude using double-altitudes.

2.1. Fixed double-altitudes

For a navigator using the Bris sextant, the limitation is that the angles cannot be adjusted. Any data must be collected in the form of the time at which a celestial body (here the Sun) passes at a fixed altitude above the horizon, ascending and descending. In this case, the times of the altitudes will be equidistant from the meridian and, with an accurate timepiece, can readily generate a longitudinal fix. However, these two times can also be used to determine latitude (Figure 3).

The time between fix 1 and fix 2 can be used to calculate an angle in the Equatorial coordinate system, following the principle of the Hour-Angle. These two times are also when the celestial body's path on a particular Equatorial declination intersects the Horizontal (Azimuthal) coordinate system at a known altitude (that of the Bris sextant). Since we know the declination of the celestial body in the Equatorial coordinate system, we have two sides (b and c in Figure 4) and an angle (C in Figure 4) of a spherical triangle; the third side (a in Figure 4) represents the angular difference between the Equatorial pole and the Horizontal pole. Solving for this distance gives us the latitude of the observer[1].

[1] following the solution proposed by Keith Brescia (AKA KBHornblower) in discussion at https://www.cloudynights.com/topic/811110-can-we-predict-the-path-of-the-sun-from-observation-points-along-the-transit/

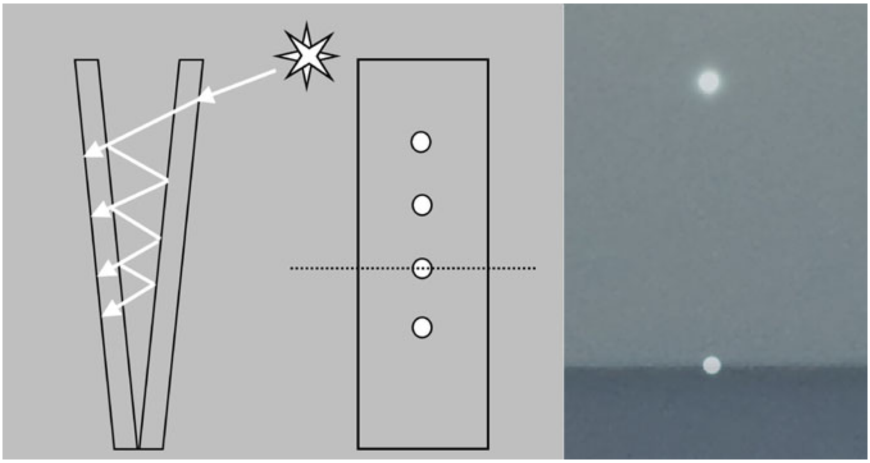

Figure 2. *Schematic of a simple version of a Bris sextant and a photograph of the refracted images. The light passes through a single plate of glass and hits the opposite plate, which acts as a beam-splitter, passing some through and reflecting some back. This is repeated, so that multiple images appear on the side of the viewer (as in right image). For more details of the calculation of the angles of the beams, see Nenninger, 2000 and Yrvind, 2008. Here a two-piece device is portrayed for simplicity; however, three is more typical to produce more visible images (lower images are dimmer as the beam gets split more times). Any image superimposed over the horizon represents a specific angle of altitude. Here the centre of the Sun is over the horizon for simplicity's sake, but the angle can also be determined using the Sun's lower limb for greater accuracy. In the right image, only two of the refracted images are in the photograph. Three more were visible to the user, although the camera could not capture them in one view. Note that the upper image is brighter – some images require the use of a filter (see Figure 1).*

Solving for this triangle follows Napier's normal spherical triangle rules for two sides and a non-included angle (using the angle C and sides b and c as in Figure 4):

$$\text{angle } B = 180 - \arcsin\left(\frac{\sin b \sin C}{\sin c}\right)$$

$$\text{side } a = 2 \cdot \arctan\left[\tan\left(\frac{1}{2}\ (b-c)\right)\ \frac{\sin\left(\frac{1}{2}(B+C)\right)}{\sin\left(\frac{1}{2}(B\ -C)\right)}\right]$$

Once side a is solved for, determination of the latitude of the observer is straightforward, using the equation $(90 - a)$. Although in this example (at equinox) there is no solar declination for which to account, in almost all real cases, the declination value (positive or negative) must be added to b. For navigators in the Southern hemisphere, the sign of the declination must be reversed (positive to negative and vice versa). Note that in the calculation of angle B, the arcsine input must be between -1 and $+1$ (see asymptotes in the solution-space calculations below).

3. Evaluating the method

It may be that the relatively complex calculations have prevented the previous development of this method, as it is cumbersome to perform by hand even with a modern scientific calculator. To develop tables without a computer would be a fatiguing task, since the length b has to be adjusted for the declination of the celestial body, requiring a unique set of solutions for every declination value. The inflexibility of the Bris sextant itself imposes several constraints on the use of this method. The most

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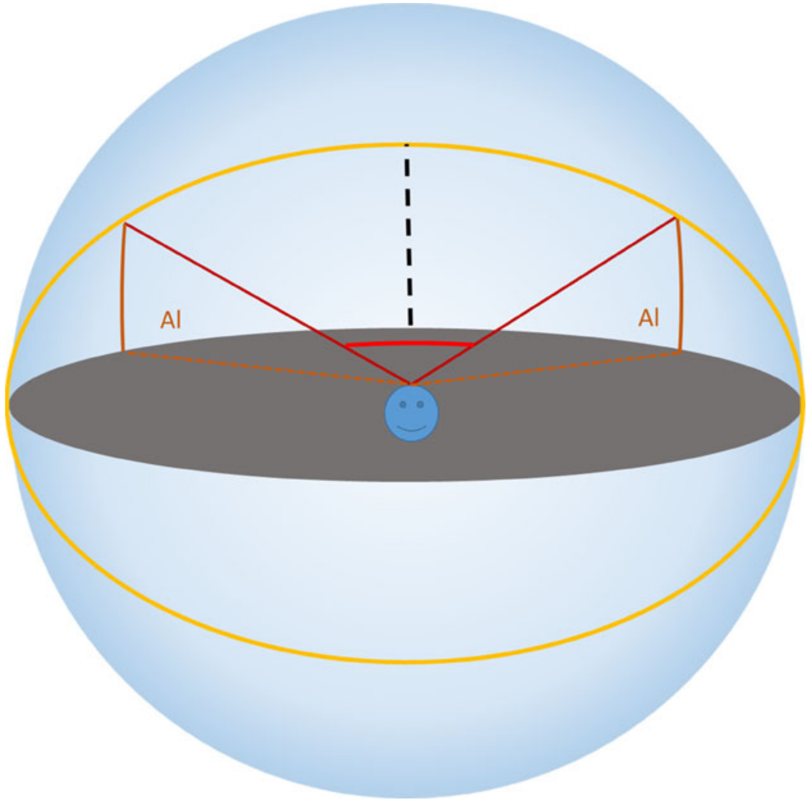

Figure 3. *The observer takes sights on the celestial body as it passes through the fixed-angle of the Bris sextant ascending, then descending. By knowing the altitude and timing of the observations and the declination of the celestial body, it is possible to derive latitude (the height of the black line) and longitude (the East–West position of the black line) using the methods proposed here.*

important constraint is that if the Sun (or other celestial body) never rises above the horizon enough to reach the Bris sextant's minimum altitude of observation, the method cannot be used.

To evaluate the value of this method, the solution space for several fixed angles was calculated [Figure 5(a)–(c)]. In this simulation, a range of 50 declinations was generated that, roughly, cover the span during a typical year (−25° to +25°). For three altitude values (20°, 40° and 60°) every value of angle *C* and the resultant latitude (over a single day) was plotted. The goal was to determine the general utility of this method, given the Bris sextant cannot be adjusted.

At some latitudes, during the solstices, using a sextant with a single altitude is useless because either the Sun did not ascend to the plane of the device's altitude or, as at very high latitudes, a single altitude provides poor resolution. However, the first problem can be mitigated by the use of the other angles on the sextant. A Bris device composed of three planes of glass can have more than eight workable angles (Nenninger, 2000), depending on the angles of the glass planes and their reflective capacities and even the two-plane model (Figure 1) produces five useful angles.

In terms of any single angle, some limitations are clear, as at any given date, the range of coverage for a single angle is from 45 degrees down to only a few (near the poles at the solstice). However, with even three angles, most of the latitudes are solved and good coverage is possible. The pattern of asymptotes indicates that the greatest resolution is achieved with lower values for angle *C*, which means taking sightings closer to the meridian, similar to traditional ex-meridian or double-altitude methods that have to be taken within an hour or 30 min of noon (Cotter, 1964). However, good resolution is attainable

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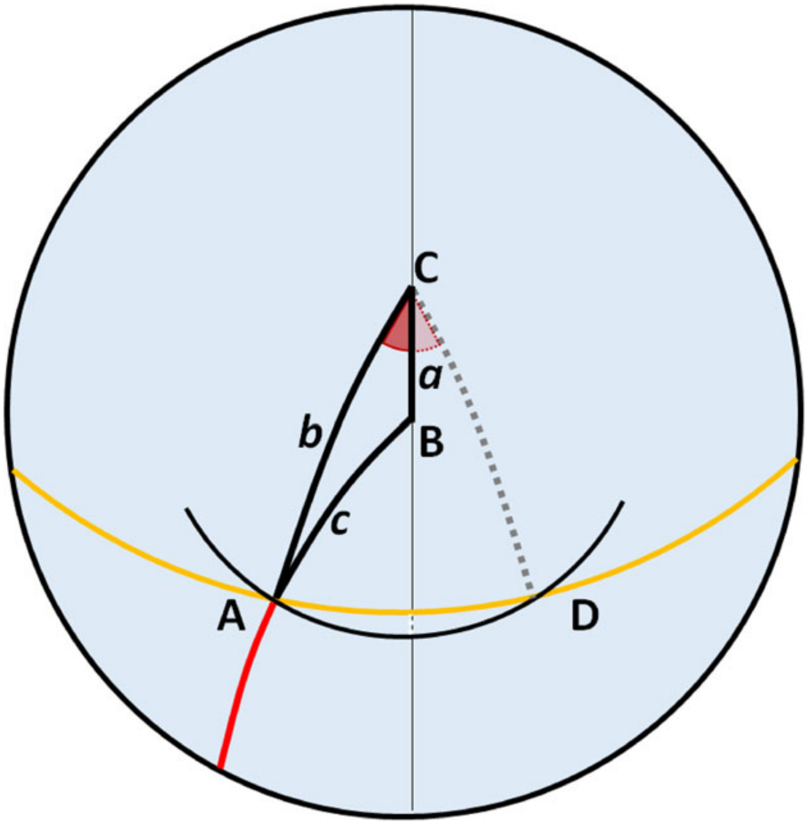

Figure 4. Spherical triangle to be solved, following the colour scheme of Figure 3 (looking straight up, with the Azimuthal pole at B and the Equatorial pole at C, and the horizon represented by the outer circle). In this figure (at equinox, for simplicity's sake), the Sun is observed at time A and time D. The altitude of the Bris sextant is the red line and the black line is the plane of the observations of the Bris device (which cross the yellow path of the Sun at A and D), so c is 90 – altitude. At equinox, solar declination is at 0, so b is 90°. At other times of the year, b will be (90° ± declination). Angle C is one half the angular difference between the two observations (points A and D). The leg to be solved is a, which gives the latitude of the observer (90 – a). Figure credit: after illustration by Keith Brescia.

well outside the traditional limit of one hour. For an observation of the Sun at 20 degrees of altitude, observations can be taken as far apart as four hours [see Figure 5(a)].

4. Using the method with a traditional sextant

The utility of this method is that the traditional nautical sextant need not be used. However, the quantitative elements certainly can be used with a standard sextant and in this case, the timing is of the user's convenience – all that is necessary is that the same reading be taken after the passing of the meridian. Advantages come with the use of a sextant, as multiple pairs of readings close together could be taken and the results averaged, which would increase the resolution considerably and reduce the potential effect of error. A simple computational application could readily be developed for any computing device (smartphone, tablet, etc.) such that a few parameters (the altitude, times of the sightings, the declination, dip) would enable the user to find longitude and latitude without the use of any satellite signal. With multiple pairs of sightings, shifts in cloud cover could be accommodated unless the sun is truly obscured all day. With an artificial horizon (using the Bris device or traditional sextant) or a bubble sextant, this

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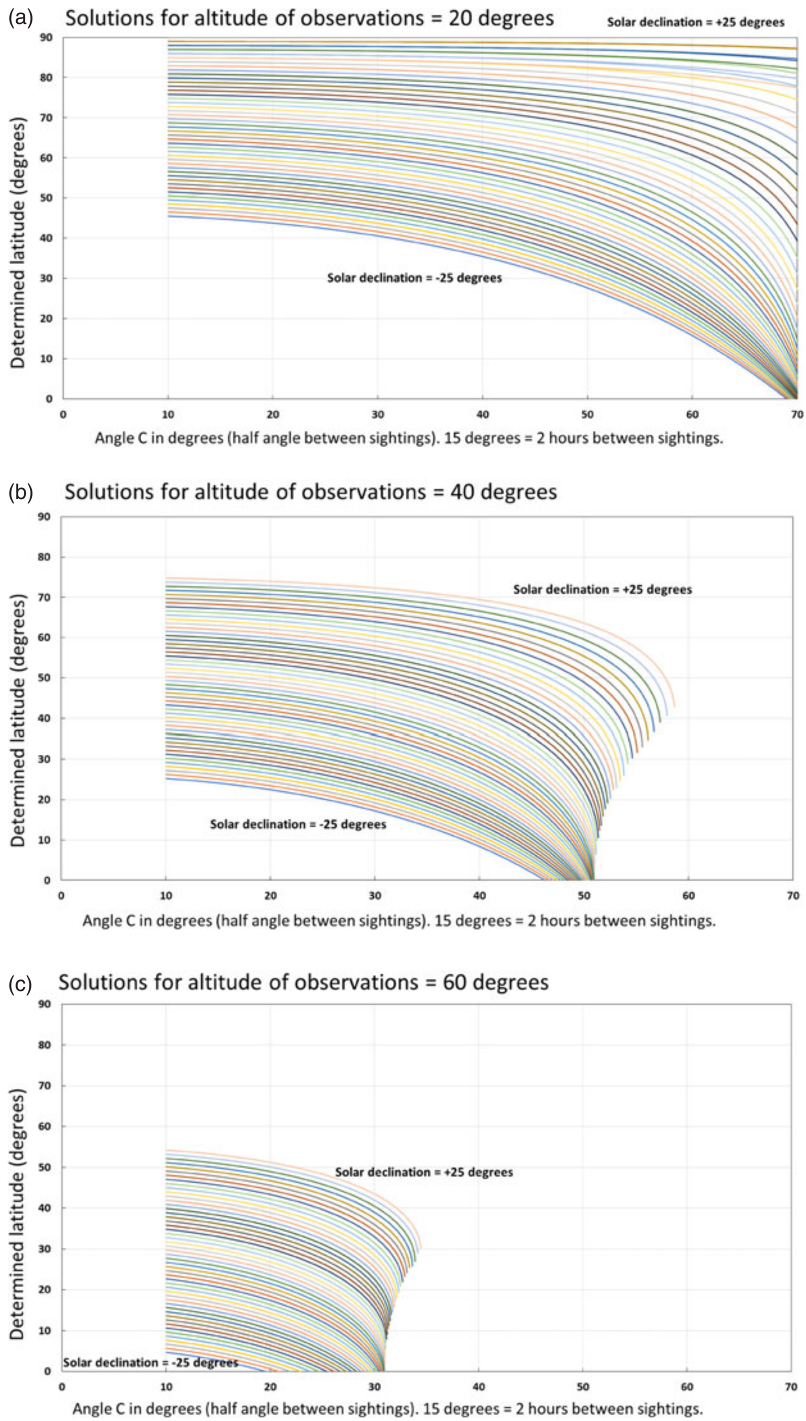

Figure 5. *(a–c) Solution space for a two-altitude sight using a Bris sextant of three different angles, over various times of the year (declinations from −25° to +25°). The X-axis represents increasing values of C (half the angular time between the two observations). The Y-axis is the latitude determined from calculation at any given X position. The different coloured lines are (descending) ranges of values for declinations, modelling the variation from summer solstice (here at +25°) descending to winter solstice (here −25°).*

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method is also appropriate for terrestrial navigation. An illuminated bubble sextant (such as a Link A-12 or the Cassens and Plath bubble attachment for a traditional sextant) would enable the use of the mathematical principles offered here on any celestial body that is moving through the night sky on the equatorial sphere, using the nautical almanac to determine the Equatorial coordinates of the object.

5. Worked example

A sighting of the Sun is taken at 10:30:36 AM and repeated at 1:16:59 PM. The altitude of observation is 40 degrees. The time zone is +8hrs from GMT. Declination for the date is −10·005 degrees. The solution is as follows:

Side b: 100·005 degrees (90 − declination)
Side c: 50 degrees (90 − altitude)
Angle C: 20·7979167 degrees (2 h, 46 min, 23 s or 2·773056 decimal hours·15°/h)/2
Angle B: 152·8407 degrees $180 - \arcsin\left(\frac{\sin 100.005\ \sin 20.7979}{\sin 50}\right)$
Side a: 54·0089959 degrees $2\ \arctan\left[\tan\left(\frac{1}{2}\ (100.005-50)\right)\ \frac{\sin\left(\frac{1}{2}(152.840+20.7979)\right)}{\sin\left(\frac{1}{2}(152.840-20.7979)\right)}\right]$
Latitude: 35·99104° or 35°59·4′ (90 − c)
Longitude: mean of 13:16:59 & 10:30:36 = 11:53:47, difference from 12:00 = −00:06:12 (−00:06:12·15°/h) + (8 h·15°/h) = 118·45° or 118°27′

6. Conclusion

The goal is to develop a method of determining location using celestial methods that eliminates the need for a comparatively expensive, fragile and complex instrument that requires considerable training and skill – the sextant. A relatively untrained user can use the Bris device more readily than a typical nautical sextant, particularly if sighting from a rocking boat. Further, the need for traditional plotting tools and workspace, typically required for identifying the location of the ‘cocked hat’, is eliminated. This method produces a result that is a specific position, using latitude and longitude, that places the user on the map with a minimum of training and monetary investment. For navigators with a traditional sextant, this method offers a rapid fix to supplement established methods. The author is currently developing a smartphone-/tablet-based app that will allow input of the various parameters (time 1, time 2, altitude of observation, declination, dip) that carries out a calculation and gives a direct readout of longitude and latitude. Additionally, dynamic tables (via Excel spreadsheet) are being developed that will allow the user to enter a Bris sextant angle, dip, and that day’s declination to produce a printable table of solutions for that day, or a series of tables over many days or weeks. Both the app and spreadsheet tables will be provided online for free at my laboratory website: https://brianvillmoare.com/projects-2.

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